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ABSTRACT

This investigation looks at the difficulties tenth-grade students have with the concept of proportionality and the problems this causes in tenth-grade physics. Students were asked to solve various problems in the areas of volume measurement and weight, all relating to proportionality, and their approaches to the tasks were recorded. As a result, the students were categorized according to Piagetian terms. (BB)

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SE 026 569

CONCEPT FORMATION IN MATHEMATICS AND THE NATURAL SCIENCES, THE INTEGRATED UPPER SECONDARY SCHOOL (5)

STUDIES OF
MATHEMATICS IN TEACHING OF SCIENCE
IN GÖTEBORG

Paper presented at the Joint ICMI/ICPE/CTS/ UNESCO/IDM-conference on 'Cooperation between Science Teachers and Mathematics Teachers', September 17 - 23, 1978, Institut für Didaktik der Mathematik, Bielefeld, FRG.

Leif Lybeck

No 72 October 1978

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FORWORD

This paper was presented at the Joint ICMI/ICPE/CTS/ UNESCO/IDM-conference, Bielefeld, FRG.

In report No 71 of this series, I have explained the meaning of what the capital letter abbreviation represent. The programme and the content of the conference is also given.

Report No 71 contains the paper on 'A research approach into Science Education at Göteborg' presented at the conference as well as an appendix which is the product of a Working Group of which I was a member.

Mölndal October 1978.

Leif Lybeck

CONCEPT FORMATION IN MATHEMATICS AND THE NATURAL SCIENCES, THE INTEGRATED UPPER SECONDARY SCHOOL is a translation from the Swedish name BEGREPPSBILDNING INOM MATEMATIK OCH NATURVETENSKAPLIGA ÄMNEN, GYMNSIESKOLAN (the BMN-project).
The BMN-project is sponsored by the National Swedish Board of Education during the fiscal years 1975/79.

## STUDIES OF MATHEMATICS IN TEACHING OF SCIENCE IN GÖTEBORG

### BACKGROUND

Eight out of ten students who left the 9-year Swedish compulsory school during 1971 continued their education in one of the lines of the integrated upper secondary school within the next 4 years. In this study, the natural science line (N) and the technology line (T) are of special interest. During the years 1971-75, recruitment to the T-line as well as the total input of students into the integrated upper secondary school was fairly stable. At the same time recruitment to the N-line decreased by about 30 %, and the so-called "flight from the N-line" to other lines, e.g. the social science line (S), decreased the rate of flow of the students through the N-line to about 65 %. These statistical facts about the school system have their counterpart in the recruitment figures to the faculties of natural sciences and the institutes of technology. 70 % of the graduated students from the N-line, and 20 % of the graduated students from the T-line continue their education at university. The flow of these students to the science faculties (mathematics, physics and chemistry) has decreased by about 60 %, and the training of graduate engineers suffers from the lack of N-students. In the school-year 1971/72 we had 10 486 students in the N-line, and 6 592 students in the T-line (both figures for grade 10, the first year of the lines). The corresponding figures for the 1977/78 are 7 289 students in the N-line, and 7 918 students in the T-line. The rate of the decreasing trend of the number of N-students has been more than compensated for by an opposite trend in the numbers of T-students during the academic years 1976/77 and 1977/78. (In the academic year 1976/77, nearly 86 600 places were available in grade 10 of all of the two-, three- and four-year lines.) At the faculties of science the number of undergraduate students per year has decreased this decade from about 4 000 to about 1 000 in mathematics, and in physics from about 900 to 130. We may add, that around 300 places on the training lines for subject matter teachers in the natural sciences at the Teacher Training Colleges were not occupied in the autumn 1977.

In an article reviewing research into science education in Sweden, I have reported on different measures taken to guard against possible causes of the trends mentioned (Lybeck, 1977). Recently, the N-group of the Board of Education has again discussed a number of problems and proposed measures (Lundberg and Norén, 1978). One step has been to elaborate new courses of study for the supplementary parts of the core curriculum for the 3-year N-line (grades 10-12, ages 16- ), and the 4-year T-line (grades 10-13, ages 16- ) at the integrated upper secondary school. An earlier course of study in mathematics led to computational skills among students that were lesser than those required in order to continue studies in physics, technology, mathematics and other subjects. The present course of study places emphasis on more time for training basic skills during grade 10 in numerical calculations and algebra, and more time during grade 10 and at the beginning of grade 11 for practising calculations with elementary functions and their graphs. The courses reinforce the earlier view that mathematics ought to be an instrument for future studies, and it lays stress upon an integration with other school subjects. The course in mathematics is common for the N- and T-lines. This is also valid for the courses in chemistry and physics. The trends in the latter courses are similar. The authors of the present courses have called attention to the conceptual difficulties the students face when studying chemistry, physics and mathematics. Two connected levels of conceptual understanding are more distictly shown to consist of problems in interpreting the objectives of chemistry and physics teaching. The levels are the students' understanding of and conception of how knowledge is produced, and how concepts are arrived at in the natural sciences; and the students' conception of concepts such as concentration, density and proportionality. In the BMN-project we are studying these aspects of the students' learning process.[1)]

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1) The project is sponsored by the National Swedish Board of Education. BMN is short for the Swedish name, and can be translated as 'Concept Formation in Mathematics and the Natural Sciences'.

Our research approach in Science Education has been discussed by Lybeck (1977, 1978 a, 1978 b). We have applied two methods in the data-generating process, namely:

(1) individual interviews with the students, and
(2) continuous recordings of the factual teaching of physics during one school-year in the experimental classes.

Two experimental classes and one control class come from the N-line, and one control class from the T-line. With the individual interviews as a basis we map the students' thought content related to the content of certain subject matters. The recordings of the students' learning processes give us a longitudinal picture of how the students' conception of some physical phenomena come into play in the teaching process.

We have produced two types of instruments for the individual interviews about the students' comprehension of some concepts in play in the factual teaching of physics in grade 10, namely:

(I) at a lower level of abstraction concerning the concepts of proportionality, density and velocity related to subject matter content, and
(II) at a higher level of abstraction concerning the students' conception of, and understanding of concepts used, such as problem, hypotheses, instrument, data, model and theory.

PROBLEM

When entering grade 10, the N-students in particular, experience a drastic change, what for many of them is a shocking shift, in how to view the world around them, as a result of the abstract world they are now facing in the teaching of chemistry, physics and mathematics. This is the main reason why we are concentrating our resources on the students of the N-line, grade 10.

The concept of proportionality is to some degree an integrating factor in the courses of study in grade 10. During the pilot studies we were able to document the fact that several of the N-students (ages 16-17) had difficulties in grasping the concept of proportionality, when the concepts of density, speed, acceleration etc were introduced. Previously, or simultaneously, the concept of proportionality was taught in mathematics lesson. Thus, it was necessary to study the students' proportional reasoning.

## METHOD

In previous studies we have found qualitatively different ways of apprehending the content of certain subject matter (Lybeck, Marton and Säljö, 1976; Lybeck, 1978 a). The set of qualitatively different conceptions are research results describing the students' thought content, as manifested in their answers in individual interviews. Arguments in favour of the applied research method in this paper, i.e. to systmatize forms of thought in terms of which the people interpret significant aspects of reality as categories of description have recently been discussed by Marton (1978).

### Subjects

The subjects are fourteen students from the natural science line (N), grade 10 (ages 16-17), and twelve students from the technology line (T), grade 10 (ages 16-17), of the integrated upper secondary school. The two experimental groups belong to control classes mentioned above concerning the larger study.

The individual interviews with the students were carried out by the author at the end of the first working month after the students' summer vacation. At the time of the interviews, the students had once more studied, and in more detail, the concept of proportionality, linear functions and graphical representations of the functional relationship, which also constituted the content of the mathematics teaching in the

upper compulsory school in grades 8 and 9. The physics teaching during the first month of grade 10, covered, the density concept, and uniform motion. As part of the teaching, the students carried out laboratory work and calculated, for instance, average speeds from distance-time graphs.

The subjects were provided with paper and pencil to be used if necessary when solving the problems posed in the questions. As the interview went on, the necessary experimental equipment was placed on the desk in front of the student.

Tasks

We were forced to limit our research to students of ages 16-17. Thus, it was considered appropriate to use tasks on proportional reasoning given to students of other ages too. During the pilot studies we found that the tasks ought to be fairly simple ones. At the same time, the tasks should be representative of the students previous and present experiences in physics teaching. In our instrument (I) mentioned above (p 3) we used two tasks reported by Suarez (1974, 1975, 1977), and Suarez and Rhonheimer (1974). In the first task, the students were asked to predict the outcome when pouring water from a thinner measuring cylinder into a thicker one, both having equally graded scales, but unequal base areas. Thus, the units were arbitrary and not expressed in any known units. In the second task, a person had loaded a spring balance with weights and read the corresponding increases in length of the spring. The measured pair of values presented in a table were to be explained by the students, when asked specific questions.

Suarez' studies on the students' conception of direct proportionality are linked to investigations by Karplus and Peterson (1970), Karplus and Karplus (1972) and Karplus, Karplus and Wollman (1974). The Karplus' Group constructed the Paper Clips Task with the figures Mr Short and Mr Tall. This task did not require familiarity with several variables and certain physical principles in addition to the proportionality schema, as is the case with the balance and

shadow tasks (Inhelder and Piaget, 1958). Suarez extended the Paper Clips Task with Herr Riese (Mr Giant), but in all essentials he and the Karplus' Group found the same response categories among the students in Zürich and San Francisco. Their data were based on the students' written responses. But Suarez wanted to study more deeply the younger students' way of reasoning. He introduced the Measuring Cylinder Task (bottom area ratio 2:3). It was given to experimental groups with average ages of 10 and 12.5 years. The analysis of the written responses gave similar response categories as in the case of the Paper Clips Task.

The Measuring Cylinder Task and the Paper Clips Task have analogous structures and the numerical skills required are the same. Suarez (1975, 1977) has not explained why the Measuring Cylinder Task is more appropriate in order to study the proportional reasoning of younger students. According to Piaget's theory the students have to conserve the water volume when pouring the water from the thick measuring cylinder to the thin one, and they have to quantify relevant variables in order to predict the outcome. During the last two decades, a lot of studies carried out within the piagetian frame of reference have demonstrated that even older secondary school students respond as non-conservers in volume tasks. Spontaneous utterances of N-students during the pilot studies supported such results. Thus, we had still more reason to use the Measuring Cylinder Task.

After the first part of the interview, either the student (S) suggests pouring water into the cylinders to find out the differences between the cylinders, or the experimenter (E) pours water into the thick cylinder always to scale line 6. At a later stage, we want to know if the students pays attention to the scale and especially the measuring line marked 6 for the quantity of water. The scales on both cylinders are not numbered. If this quantitative aspect is not clearly thought of by the students, it is pointed out at this stage of the interview. The students now know the pair of measured numbers (6,9) when pouring 6 → 9.

The two central problems of the Measuring Cylinder Task

Question 4-y is as follow:

E: If I pour up to 4 in the thick cylinder, how high would it come in the thin cylinder (which is empty)?

The answer is followed up:

E: How did you work it out?

When the students had answered, they were asked to check their prediction experimentally, if they had not done so spontaneously. If they found necessary, they were permitted to correct their reasoning.

Question 9-y follows the same procedure as in question 4-y. Suarez' problem in the Paper Clips Task with Mr Giant (Herr Riese) is analogous to question 9-y. It demands slightly more numerical skills to be solved correctly.

And now to the second task, the Spring Balance Task. In order not to complicate the proportional reasoning problems with too much physics we pointed out that the dynamometer could be used to weigh objects. If so, it is called a spring balance, as in the written task, the student was then asked to read before answering some questions. (In fact several of the students who live on the Swedish West Coast had seen the instrument being used by fishermen.) On the sheet of paper the student sees a picture of an unloaded and a loaded spring balance (see fig.3, p19), The measurements are given in a table.

| Lengthening | Weight |
|---|---|
| 4 mm | 20 g |
| 10 mm | 50 g |
| 20 mm | 100 g |
| 50 mm | 250 g |

The central problems of the Spring Balance Task

The following questions were written on a sheet of paper.

Question 100g-20mm: It appears from the table that the length of the spring is increased by 4 mm when the 20 g weight is hung on it. Can you

explain why the spring is lengthened 20 mm, when the 100 g weight is used?

Question 80g-y: How much is the spring lengthened when a 80 g weight is attached?

Question 140g-y: How much when a 140 g weight is attached?

Those questions are borrowed from Suarez (1974, 1975, 1977). We added three oral questions.

Question 90g-y: How much when a 90 g weight is attached?

Question 115g-y: How much when a 115 g weight is attached?

Question 123g-y: How much when a 123 g weight is attached?

The last three questions make great demands on the students' calculative skills, if they do not use the general solving method, e.g. in the form $y = k \cdot x$. The purpose is to force the student to apply the general solving method.

RESULTS

After the collection of data the recordings were transcribed and the transcriptions were checked against the tapes by the author, who carried out the interviews. A very time-consuming, qualitative analysis of the students' responses was then initiated in order to find out how the N-students and the T-students had understood and solved the tasks on proportional reasoning. We soon found that our data did not fit the response categories introduced by the Karplus' Group and by Suarez. There is no algoritmic procedure for finding the distinct ways in which the students comprehend the tasks. The students' response categories can not be produced, they have to be found in the very analysis of the data of the students' thought content related to the content of the subject matter in the tasks. This experience is shared by our colleagues (Marton and Wenestam, 1978).

Firstly, the students' answers have a common thought dimension mirroring the concept of function, and secondly, their answers have an interacting thought dimension expressing a conception of how to quantify. The first dimension we have called the functional aspect, which contains the categories A and B. The second dimension is named, the quantitative aspect, and

has for possible categories 1-4. The last aspect shows what kind of mathematical instrument the students applied in their calculations. We shall now describe the two-dimensional shema of response categories, firstly, along the functional aspect, and after that along the quantitative aspect. We shall start with the main results on the Measuring Cylinder Task.

The functional aspect, the Measuring Cylinder Task

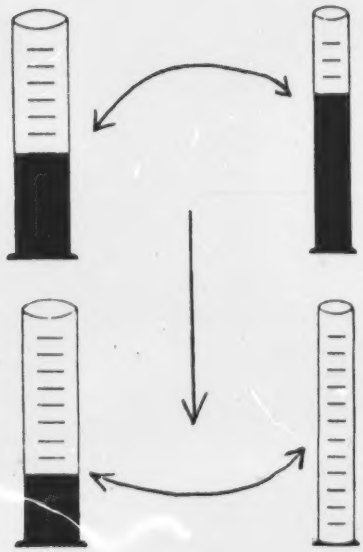


Fig. 1

A S keeps the volume constant. A change of the height between the two cylinders for one volume gives a corresponding change of the height between the two cylinders for another volume. This is the qualitative aspect.
The problems are solved in two steps:
1 The relation between the height for the first volume is quantified.
2 The relationship is used for the next volume.

We can describe category A symbolically as follows:

S 6N (the student number 6) gave an answer representing the category A:

Our algebraic interpretation of the oral response.

...

S: So I took how many times 6 goes into 9. ( $\frac{y}{x} = \frac{9}{6}$ (=1.5))

E: Yes

S: So I get ... Yes, then I get a value of how many times more, how many markings there'll be in that one in relation to this one.

E: Yes. In the thin cylinder in relation to the thick one?

S: Yes, and it is 1.5.

E: I see! And then?

S: Yes, after that I took 1.5 times 4, it makes 6. ($y = 1.5 \cdot x$)

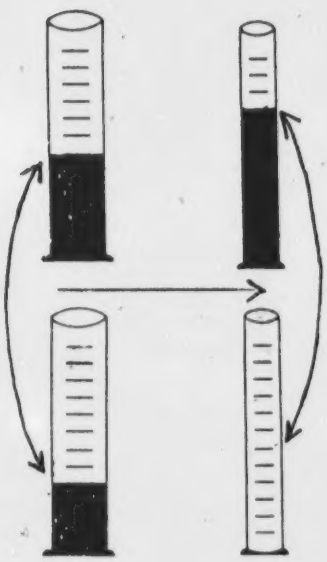
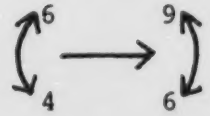


Fig. 2

B Firstly, S consider one of the cylinders. A change in the height (a change of the volume) within one of the cylinders gives a corresponding change in the height in the other cylinder.
This is the qualitative aspect.
The problems are solved in two steps:

1 The relation between the heights in the first measuring cylinder is quantified.
2 The same relationship is used for the second cylinder.

Symbolically we describe category B like this:




As an example of that solving strategy we show the response of S 1N:

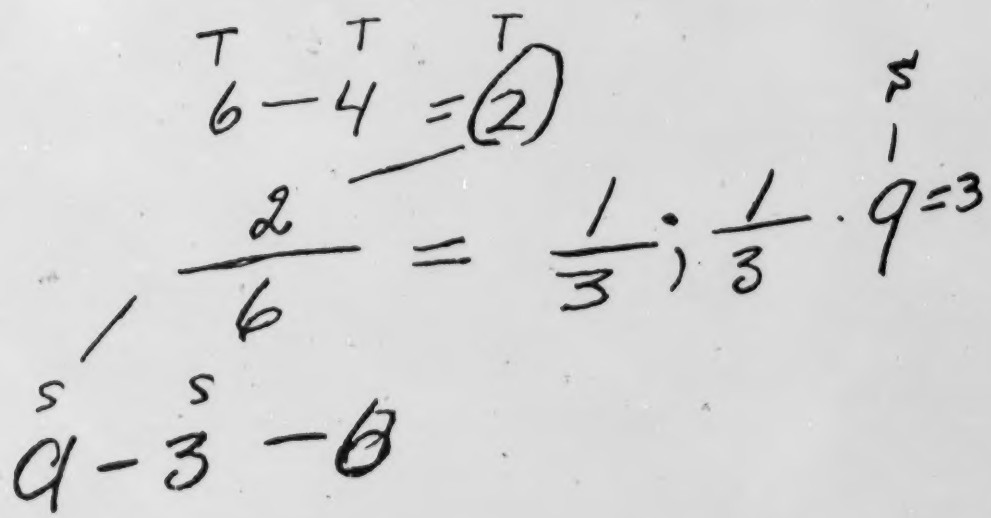
Our algebraic interpretation of the oral response.

E: To 6. How did you count it?

S: 2 over 6 is equal $\frac{1}{3}$, and 9, yes, $\frac{1}{3}$ times 9, it's 3, then. Then one takes 3 minus 9, no 9 minus 3. 6.

$(\frac{x - y}{x} = \frac{1}{3})$

$(y = x - \frac{1}{3} \cdot x)$




(The notation done by S 1N. T for the thick cylinder and S for the thin one or small one as S 1N called it.)

It is clear that S 1N gave the difference between the heights in the thick cylinder (6 - 4 = 2), as a part of the first given height (6). This fraction was used for the thin cylinder. Even here the difference is $\frac{1}{3}$ of the first height (9).

Within the categories A and B of the functional aspect, the students expressed the changes in different ways. In fact, we found that the students applied the same mathematical instrument independently of A and B. The analysis resulted in four categories when the students were quantifying the relationships.

The quantitative aspect, the Measuring Cylinder Task

1 Direct proportion. The relationship is expressed as a multiplication or a division or as a relation (ratio), e.g. as in

$$y = 1.5 \cdot x \text{ or } \frac{y}{x} = \frac{3}{2}$$

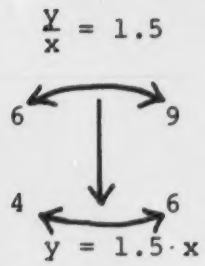
or inverted as in

$$x = \frac{y}{1.5} \text{ or } \frac{x}{y} = \frac{2}{3}.$$

S 6N quoted above is an example of category A, as S 10N:

| | | Our algebraic interpretation of the oral response. |
|---|---|---|
| S: | I divided 9 by 6. | $(\frac{y}{x} = 1.5)$ |
| E: | Yes. | |
| S: | And I multiplied what I then got, 1.5 by 4. | $(y = 1.5 \cdot x)$ |

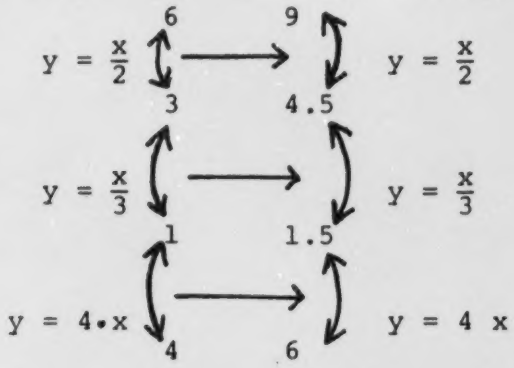
On the right hand side of the quotation is shown our classification of the thought forms interpreted with the aid of algebraic forms. We use x and y in the conventional sence as first and second term in the pair of values (x,y). Notice that the students S 1N and S10N used such a form of the relationship which explicity expresses the number they wanted to reach. We can describe S 10N's solving strategy like this:



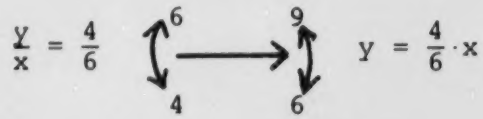

In order to demonstrate that the way to quantify is applied in both of the categories A and B of the functional aspect, we quote S 9T (T-student number 9) from category B:

| | | Our algebraic interpretation of the oral response. |
|---|---|---|
| E: | ...and there you write 3 and 4.5. Why do you do that? | |
| S: | The half. | $(y = \frac{x}{2})$ |
| E: | ...(Question 4-y)..., if it is 4 in the thick cylinder | |
| | | $(y = x \cdot 1.5)$ |
| S: | 6. | |
| E: | You got 1.5? | |
| S: | Yes. | |
| E: | How did you get it? | |
| S: | Divided 3 by 4,5. (Note the order!) | $(y = \frac{x}{3})$ |
| E: | Oh! Why did you divide by 3? | |
| S: | In order to get how much it is per step. | |

Symbolically S 9T's solving strategy is as follows:




S 9T used the way of quantification in several steps and does not directly approach the problem like this:




He introduced two hypothetical volumes. This seems to be connected with the difficulties a multiplication by $\frac{4}{6}$ implies. The demonstrated pattern in the students' way of reasoning occurs much more frequently in the Spring Balance Task, because those items have figures that did not produce any problems of quantifying directly.

From the perspective of Mathematics and Science Education, we wish to emphasize the essential different thought forms in the response of the students S 1N and S 10N on one hand, and S 9T on the other hand. Category A of the functional aspect brings the student to the algebraic form $y = 1.5 \cdot x$, in this case, the general functional relationship of the direct proportionality in the studied task. And category B implies quantitatively algebric forms, which do not express any general direct proportionality, even if the form used is $y = x \cdot 1.5$. This fact must be kept in mind when concepts based on direct proportionality are produced in the concept formation processes in science teaching.

2 Proportional increase and decrease. The relationship is expressed as an addition, subtraction or difference with the aid of an (increment) factor or ratio, e.g. as in

$y = x + \frac{1}{2} \cdot x$, $y - x = \frac{1}{2} \cdot x$ or $\frac{y - x}{x} = \frac{1}{2}$

or inverted as in

$x = y - \frac{1}{3} \cdot y$, $y - x = \frac{1}{3} \cdot y$ or $\frac{y - x}{y} = \frac{1}{3}$

We quote the response of S 9N:

S: Or... yes, at 6 in the thick cylinder, it made 9, when poured over. So it is, of course, one adds the half of 6. ($y = x + \frac{1}{2} \cdot x$)

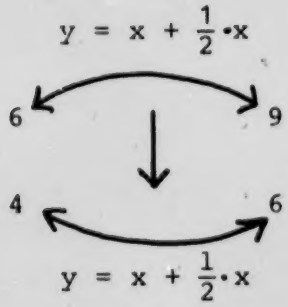
E: I see!

S: And as it stood at 4 there, so I add 2, so it'll be 6 ($y = x + \frac{1}{2} \cdot x$)

E: I see! And you think it is correct?

S: Yes, I think so.

As we can see S 9N is quantifying between the measuring cylinders:




Her response is thus classified in category A. But we have found the same way of quantifying for category B responses (the student S 1N):

S: Yes, it stood 6 before I thought.

E: Yes.

S: Now it has decreased by 2. It is, of course, $\frac{1}{3}$ of 6. And $\frac{1}{3}$ of 9 as it was then (question 6-9), it is, of course, or it will be $\frac{2}{3}$, it is 6. ($y = x - \frac{1}{3} \cdot x$) ($y = \frac{2}{3} \cdot x$)

E: Well...

S: Eh. 6 minus 4, that makes 2. ($x - y = \frac{1}{3} \cdot x$)

E: Yes.

(The rest of the response is quoted on page 11.)

Symbolically we describe his solving method like this:

$$\frac{x - y}{x} = \frac{1}{3} \quad \begin{matrix} 6 \\ 4 \end{matrix} \longrightarrow \begin{matrix} 9 \\ 6 \end{matrix} \quad y = x - \frac{1}{3} \cdot x$$

Firstly, S 1N expresses the relationship with the aid of a decrease ($y = x - \frac{1}{3} \cdot x$). Writing down his thoughts (p 11), he expresses $\frac{1}{3}$ explicitly in the form $\frac{x - y}{x} = \frac{1}{3}$. When using the relationship in the other cylinder, he changes from the direct proportion ($y = \frac{2}{3} \cdot x$) to proportional decrease ($y = x - \frac{1}{3} \cdot x$), but at the end he applies the latter form. In both cases the unknown is explicitly expressed.

3 Relating either absolute increase, absolute decrease, or absolute difference. The student realizes that the increase, decrease, or difference cannot be absolute (see category 4 below), but the student does not succeed in quantifying the relationship as it is understood in the categories 1 and 2 (above).

As an example of this we quote a category A response, (the student S 6T):

S: ...7 or... No, not quite 7, close to 7, I think... 6,5 or something like that.

E: Nearly 7? Why do you think so?

S: Well, I don't know.

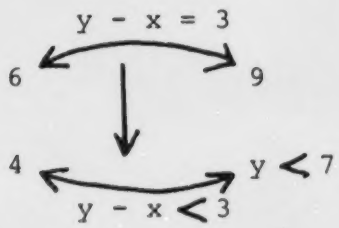
E: ...Can you tell me how you thought?

S: Eh..., if 6 and 9, so it was 3 apart. 4 is less, and then it can, of course, not be 3 apart. ($y - x = 3$)

E: I see!

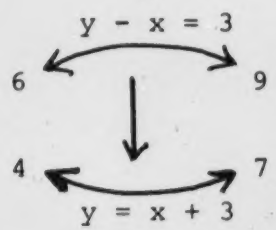
S: It won't be 3, as much now,
it should be 7, that. ...
Yes. So it can't be quite 3
now, when it is less. $(y - x < 3)$

The solving strategy is described as follows:




4 Quantification through absolute increase, decrease or difference. The relationship is looked upon as an addition, subtraction or a difference as in
$y = x + 3$ or $y - x = 3$
or inverted as in
$x = y - 3$

Symbolically the student S 5N operates between the measuring cylinders like this:




S: Yes, I thought like this, that
the difference between 6 and 9,
I thought then, $(y - x = 3)$
and so "plussing", adding ...
you add it, the difference
then to 4. $(y = x + 3)$

We also find this way of quantifying among the students, who have decided to operate within the measuring cylinder (category B):

E: Where must it stand, in the thin cylinder?

S: 7.

E: How did you get it?

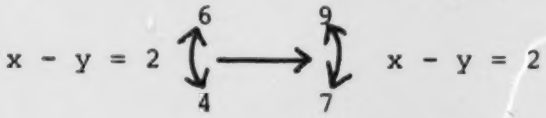
S: 6 minus 4 is 2. $(x - y = 2)$
9 minus 7 is 2. $(x - y = 2)$

E: I see!

S: It can't be correct.

(S 4N is not able to explain why.)

The solving strategy we interpret like this:




We wish to emphasize the fact that the student S 4N does not change form, when he uses the relationship in the second cylinder. Thus, he does not explicitly express the unknown magnitude.

In the relatively condensed description of the two-dimensional shema of response categories we have given a few examples of the variation within each response category. We have not described the variation of the students responses as regards the question 4-y and 9-y. We only refer to the analysis reported by Johansson and Lybeck (1978). And now we will present some results on the Spring Balance Task.

The functional aspect, the Spring Balance Task

The students' answers were classified analogously, as the responses of the Measuring Cylinder Task. A difference between the tasks is the value table (p 7). The values given are such that the students are able to relate different weights as well as different increases in the length of the spring to each other by means of simple multiples. In addition, the proportionality between the magnitudes is numeri-

cally easier to handle than in the Measuring Cylinder Task. These differences between the tasks have implied a considerably greater variation, above all in the response category B2 for the Spring Balance Task.

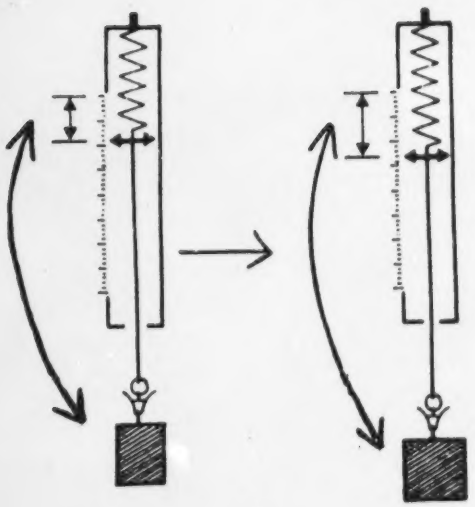


Fig. 3

A S uses one object, it means that the weight is kept constant. A change of numerical value between the weight and the increase in length of that object gives a corresponding change between the weight and the increase in length for another object. This is the qualitative aspect.
The problems are solved in the following steps:

1 The relation between the weight and the increase in length is quantified.
2 The relationship is used for the next weight.

For the question 100g-20mm the relations can be described as follows for category A:

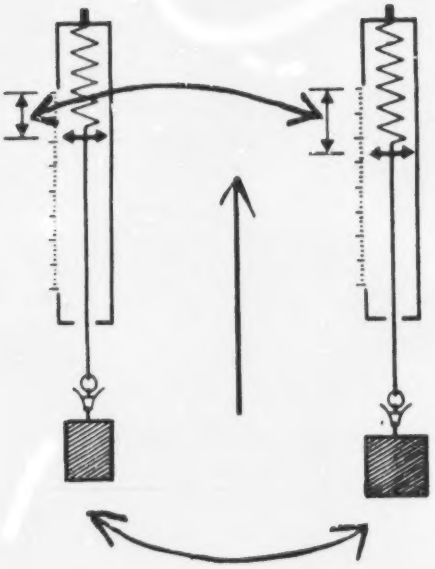
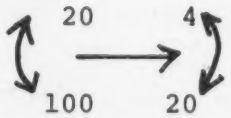


Fig. 4

B Firstly, S consider one weight at a time. A change of weight gives a corresponding change <u>in</u> the increase in length.
This is the qualitative aspect.
The problems are solved in the following steps:

1 The relation between the weights are quantified.
2 The same relationship is used between the increases in length of the spring.

Symbolically, the category B is given for the <u>question 100g-20mm</u> like this:




<u>The quantitative aspect, the Spring Balance Task</u>

This task is distinguished from the Measuring Cylinder Task by the fact that the relevant variables are of different magnitudes (in the latter task the students only considered heights). Evidently, this is the reason why none of the students have responded according to the possible category 4 as suggested by the Measuring Cylinder Task. This fact has implied three categories similar to the categories 1-3 in the Cylinder Task.

1 <u>Direct proportion</u>. The relation is expressed as a multiplication or as a division, or as a relation (ratio), e.g. as in

$y = 5 \cdot x$ or $\frac{y}{x} = 5$

or inverted as in

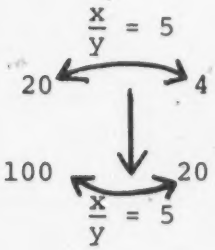
$x = \frac{y}{5}$ or $\frac{x}{y} = \frac{1}{5}$ .

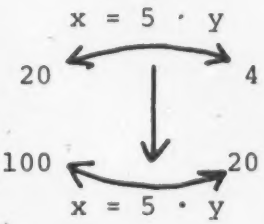
(In category A we use x for the weight and y for the increase in length of the spring, as the items concern how much the spring is stretched by different weights.)

We quote S 6N's response to question 100g-20mm in category A:

| | Our algebraic interpretation of the oral response. |
|---|---|
| S: Because, that 4 goes 5 times into 20. 10 goes 5 times into 50. 20 goes 5 times into 100, and 50 goes 5 times into 250. | ($\frac{x}{y} = 5$) |
| E: You are comparing, 4 mm with 20 g then? | |
| S: Yes, that's it. I'm comparing that column with that one. | |
| E: I see! And then you got, that...? | |
| S: Thus, 5 times the number of mm gives the weight. | ($x = 5 \cdot y$) |

Symbolically we interpret the thought forms like this:

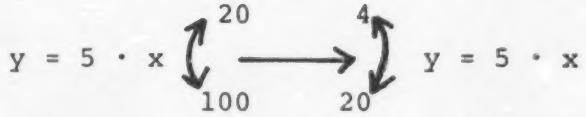



S 10N used the same strategy as S 6N, but within category B as follows:

| | |
|---|---|
| S: And 100 g, it is 5 times as much as 20 g. | |
| E: I see! | |
| S: Thus it ought to be 5 times as much as 4 mm, thus 20 mm. | ($y = 5 \cdot x$) |

Symbolically:




We recognize that S 6N's thought form according to category A gives him a general description, and thus he did not need to give the relationship for any special weight. The student S 10N did not reach any general description due to the category B strategy. "The 5" is valid only when answering question 100g-20 mm. In fact, S 10N specified the particular length, 4 mm, for which the relationship is valid. The reader has perhaps observed, that S 10N answered with a category A1 answer for the Measuring Cylinder Task. In the Spring Balance Task he switched between B1 and A1 starting from the fourth item.

2 Proportional increase or decrease. The relation is expressed as one or several additions, subtractions, or differences with the aid of one or several (increment) factors.

For a category B2 respons to question 80g-y the student S 12T answered:

S: Firstly, I took 50 g, it is 10 mm, and after that 20 g is 4 mm, and so 10g, 2 mm.

E: So, 10 g is 2 mm? How did you get it?

S: If it is also 1/5 of...yes, the half ...10 g is the half of 20 g.

E: Yes.

S: And 4 mm, and 2 mm is the half of 4 mm.
...
2 mm but I found a better way.

E: How?

S: To take 20 g minus 100 g.

E: Well, well, that is another way?

S: And 20 minus 4.

It is quite clear that S 12T divides 80 g like this in parts:

$$80 = 50 + 20 + \frac{1}{2} \cdot 20.$$

He makes use of several references. He could have used 50 g for the increase, e.g. as in

$$80 = 50 + 0.6 \cdot 50$$

or in algebraic form

$$y = x + 0.6 \cdot x,$$

but he applies the increase in a simpler form with the aid of the value table

$$30 = 20 + \frac{1}{2} \cdot 20$$

or in algebraic form

$$y = x + \frac{1}{2} \cdot x.$$

It is evident in the response he could have expressed 80 g like this

$$80 = 100 - \frac{1}{5} \cdot 100$$

or in algebraic form

$$y = x - \frac{1}{5} \cdot x.$$

But he expressed the decrease explicitly

$$80\ g = 100\ g - 20\ g.$$

When applying this relationship for the increase in length or the spring he had no problems:

$$y = 20 - 4.$$

We interpret his solving strategies for <u>question 80g-y</u> as follows:

$$y = x + x_1 + \frac{1}{2} \cdot x_1 \quad \begin{matrix} 50 \\ \updownarrow \\ 80 \end{matrix} \longrightarrow \begin{matrix} 10 \\ \updownarrow \\ 16 \end{matrix} \quad y = x + x_1 + \frac{1}{2} \cdot x_1$$

and then:

$$y = x - 1 \cdot x_1 \quad \begin{matrix} 100 \\ \updownarrow \\ 80 \end{matrix} \longrightarrow \begin{matrix} 20 \\ \updownarrow \\ 16 \end{matrix} \quad y = x - 1 \cdot x_1$$

3 Relating either absolute increase, absolute decrease or absolute difference. The student realizes that the change cannot be absolute, but the student does not succeed in quantifying the relation in the sense of the categories 1 and 2 above.

For the question 90g-y the student S 4N responded within the category B like this:

S: 19 mm then.

E: How do you calculate?

S: 50 g it stretches 10 mm, and 100 g it stretches 20 mm (pair given in the value table). It becomes 19 mm?

...

E: Are you quite sure of 19 mm?

S: It can't be correct. It becomes 19. Well, I say 19 mm. It is the only one I can get in this case.

Although there are a lot of detailed results we must, at this point, end our presentation of the empirical findings based on the Spring Balance Task and the Measuring Cylinder Task. In table 1 we present the distribution of the frequencies of the dominating thought forms concerning the Measuring Cylinder Task, and in Table 2 for the Spring Balance Task as well.

Table 1

FREQUENCIES OF DOMINATING THOUGHT FORMS, THE MEASURING CYLINDER TASK

Question 4-y

| | | Response categories within the quantitative aspect | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | Σ |
| Response categories within the functional aspect | A | 7 | 9 | 2 | 4 | 22 |
| | B | 2 | 1 | - | 1 | 4 |
| | Σ | 9 | 10 | 2 | 5 | 26 |

Question 9-y

| | | Response categories within the quantitative aspect | | | | |
|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | Σ |
| Response categories within the functional aspect | A | 5 | 9 | 5 | - | 19 |
| | B | 4 | 2 | 1 | - | 7 |
| | Σ | 9 | 11 | 6 | 0 | 26 |

Table 2

FREQUENCIES OF DOMINATING THOUGHT FORMS, THE SPRING BALANCE TASK

Question 100g-20mm

| | | Response categories within the quantitative aspect | | | |
|---|---|---|---|---|---|
| | | 1 | 2 | 3 | Σ |
| Response categories within the functional aspect | A | 10 | - | - | 10 |
| | B | 14 | - | 2 | 16 |
| | Σ | 24 | 0 | 2 | 26 |

Question 80g-y

| | 1 | 2 | 3 | Σ |
|---|---|---|---|---|
| A | 9 | - | - | 9 |
| B | 12 | 4 | 1 | 17 |
| Σ | 21 | 4 | 1 | 26 |

Question 140g-y

| | 1 | 2 | 3 | Σ |
|---|---|---|---|---|
| A | 9 | - | - | 9 |
| B | 7 | 8 | 2 | 17 |
| Σ | 16 | 8 | 2 | 26 |

Question 90g-y

| | 1 | 2 | 3 | Σ |
|---|---|---|---|---|
| A | 9 | - | - | 9 |
| B | 1 | 11 | 4 | 16 |
| Σ | 10 | 11 | 4 | 25 |

1 subject was unsuccessful

Table 2 (continued)

FREQUENCIES OF DOMINATING THOUGHT FORMS, THE SPRING BALANCE TASK

Question 115g-y

| | 1 | 2 | 3 | Σ |
|---|---|---|---|---|
| A | 8 | - | - | 8 |
| B | 1 | 12 | 3 | 16 |
| Σ | 9 | 12 | 3 | 24 |

2 subjects were unsuccessful.

Question 123g-y

| | 1 | 2 | 3 | Σ |
|---|---|---|---|---|
| A | 10 | - | - | 10 |
| B | 1 | 11 | - | 12 |
| Σ | 11 | 11 | 0 | 22 |

4 subjects were unsuccessful.

## Classroom studies

As mentioned in the introductory part of this paper, we have done a lot of recordings of the factual physics teaching. In one of the experimental classes, the teacher introduced the concept of density to N-students, grade 10, by letting them measure the masses and the volumes of a number of rectangular parallelepipeds. The students tabulated their pairs of values and drew graphs. Because the objects consisted of two kinds of wood, the students, after some discussion, got two lines. Some days later, they discussed their results and the lesson ended with the following problem: The teacher faced the student with two apparently equally large boxes floating in water, but one of the boxes floated deeper than the other one. A heated discussion among the students was recorded, and, in fact, some qualitatively different conceptions of the Archimedian Principle were expressed by the students. They quantified on different conceptual levels. We shall only quote one of the students, because his rule of quantification is related to the thought form we have described in the proportional reasoning tasks,

Teacher: They want you to repeat.

Tor: Yes, but the piece of the box, which is below the water surface, it has a certain volume then, say 3 $cm^3$. And then there are 3 $cm^3$, water, which has been displaced. And then they become 3 g water too. And then that piece which isn't below the water (surface). (Note his B-form of the density concept.)
One has to calculate a percentage, how much of the piece which is below the surface in order to get the whole, and then it weighs, thus, 3 g, the piece below the water. And then, the rest...
Then one knows what a part of the box weighs, and then one only has to know how large part of it as one has weighed.

E: May I ask? If the part which is below the water surface is 1/3 of the whole (box).

Tor: Yes.

E: It weighs 3 g, you said?

Tor: Yes.

E: How much does the box weigh?

Tor: Then it weighs 9 g, then, yes.

It was interesting to observe the great interest among the students. Tor had obviously raised a hypothesis worth considering. In fact, Tor had to fight for it, and he refused to abandon his hypothesis until he and some of the other students had designed experiments, which solved the cognitive conflict. Tor thought he could weigh the part of the box beneath the surface, because that part of the object interacted with the water. Thought forms according to the category B of the funcional aspect dominated his thinking. And in fact, Tor solved the proportional reasoning tasks according to B1 at the end of the school year. The results given here have been reported by Lybeck, Marton and Säljö (1976) and Lybeck (1978 b).

Discussion

In the history of science, the concepts of proportion, ratio and proportionality have been made use of when many scientific concepts were elaborated quantitatively, and finally settled. Let us bear in mind how Gallileo expressed his functional relationships in words, and in the language of proportion (Kline, 1972).

> "The areas of two cylinders of equal volumes, neglacting the bases, bear to each over a ratio which is the square root of the ratio of their lengths."
>
> "The times of descent along inclined planes of the same height, but of different slopes are to each over as the lengths of these planes." (p 338)

In the sense of our description category B, Gallileo gave the physical laws as functional relationships.

In fact, this pattern of reasoning dominated the mathematical and physical thinking up to the seventeen century. Newton created his "method of fluxion" using ratios in the meaning

of category B. He used the term "fluent" to represent any relationship between variables. We can observe the close relation between the development of mathematical concepts and physical phenomena. The point and lines on Newton's graphs move, this idea is an old theme in the history of mathematics and science. Leibniz introduced the word "function" and used it to mean quantities that depend on a variable (Kline, 1972). We may add here, that Leibniz was the first mathematician of prominence to use systematically the dot for multiplication and to write proportion in the modern sense a:b = c:d. Thus, some kind of change in the mathematical and scientific thought pattern occurred distinguishing it from older ones, at least in the world of science.

It may be of interest to note, that Lunzer and Pumfrey (1966) in an often-quoted paper on the understanding of the concept of proportionality, themselves say

> "for we know that if a/b = c/d then a d = b c and a/c = b/d".

Yes how do we know that? In fact, their tasks of Cuisenaire rods is similar to our Measuring Cylinder Task. According to our description categories, they discussed B1 and B2 responses.

In a recently published text-book for N- and T-students, grade 10, fitted to the new course of study in physics, the authors (Staffansson, Andersson, Johansson and Snaar, 1978) introduced the concept of density as follows:

> "Our experiment tells us perhaps that 6.0 $cm^3$ of aluminium weighs 16.2 g. A half as large a volume, 3.0 $cm^3$, has a mass half as large, 8.1 g; for a three times as large a volume. 18.0 $cm^3$, the mass is three times as large, 48.6 g, and so on. If the mass increases in this way we say that the mass is proportional to the volume. It is written
>
> $m \propto V$.
>
> The relationship between the mass and the volume can be expressed by the equation
>
> $m = \rho \cdot V$."

Thus, the authors make use of a B1 thought form and present the physical law (the functional relationship) in an A1 form. In fact, the same procedure was followed when introducing the dynamometer. (We used the Spring Balance Task in our study.)

As an example of didactical phenomenology, Freudenthal (1978) has discussed the concepts of ratio and proportion. Starting from uniform motion, he says that

> "In equal times, equal distances are covered" is a popular definition; and this is, if continuity is tacitly assumed - as it should be - equivalent to the formally stronger statement "distances are in proportion to times".

This kind of ratio formed within a system is designated as internal (Freudenthal, 1978). Our description category B represents relationships of that kind. External relationships are of category A types, e.g. constant speed is given as $\frac{s}{t}$ = = const. Thus for uniform motion we have the thought forms:

$s_1 : s_2 = t_1 : t_2$ (category B1)

$s_1 : t_1 = s_2 : t_2$ (category A1).

Freudenthal says: "Interchanging the middle terms in a proportion is so familiar to us that we can hardly realise the width of this mental jump." We agree with our empirical findings concerning proportional reasoning as a basis (we have in fact data on uniform motion recorded in classroom). "Nobody raises the question whether this jump could not be too big for the learner (Freudenthal, 1978)". Our work is a contribution to find the "length" of the jump.

## SOME CONCLUDING REMARKS

1 A major working hypothesis based on our studies is the following:
The conceptual difficulties faced by the students are contextual ones,
i.e. depending on the content,
rather than structural ones,
i.e. depending on general thought structures, which in themselves are independent of the content. (Lybeck, Marton and Säljö, 1976; Lybeck, 1978 a, 1978 b).

Our empirical results support the context and content dependencies of the conceptual difficulties. Thus, some students, who are formal thinkers in the sense of Piaget's theory (Inhelder and Piaget, 1958), should be able to understand tasks on direct proportionality. Our response category A1

describes such formal thought forms. Considering the selection mechanism before our groups of students entered the N- and T-lines, and the very simple tasks they answered, they ought to have given A1 responses. But an A1 response for one of the tasks does not imply an A1 response for the other task, or even for the items within the tasks.

2 The category A of the functional aspect involves a thought pattern, where the relation between the relevant variables based on separate pair of values can be quantified in a generalized form in the functional relationship $y = k \cdot x$. The category B gives relations between individual quantities within one variable at a time. If the students are operating according to the category B, it brings with it a limitation of the other thought forms including the potential of a general relation. In fact, the student does not find any general constant. The method of quantification can formally be looked upon as being the same in the categories A and B, but in A the student gets one unknown (y), while in B the student always gets two unknowns ("k" and y). The functional thinking in category B is not the "species" of functional thinking, which leads to the concept formation aimed at in science teaching (the physical laws are presented in the text-books like this, e.g. $m = \varrho \cdot V$, $s = v \cdot t$ etc). It is the quantification of the relation between qualitatively different variables (category A1), that produce the fertile functional relationship. Also, it gives the desired simplicity in the abstract description of the world around us, which is a characteristic of modern scientific perspectives. And, in fact, it has not yet invaded the world of primitive science.

3 Against the background of our findings: How shall the students be helped to understand that it is exactly the algebraic simplicity of the direct proportionality, which is fundamental to several of the scientific concepts?

The thought forms in category B are sometimes very complicated, and we are fascinated by the richness of the students thought content as related to the tasks. But are those thought forms put into play in the teaching process? According to the working hypothesis (point 1 above), and what we have observed

empirically, the students do not see the reason to connect the different thought forms, e.g. the teaching of the concept of proportionality had not been felt useful when solving the tasks. Thus, we propose that the thought forms become a concrete part of the students' educational content. But as far as we can understand, a lot of systematic qualitative studies covering the content of the education will have to be carried out.

4 Our description categories are based on what we have called thought forms orally expressed by the students. These forms have their counterparts in knowledge forms, known from the history of mathematics and science, and of course, present day thinking in these fields. We have traced jumps in the students' understanding of a qualitative nature analogous to changes in thought patterns in the past. Thus studies of the theory of science and the history of science and mathematics can provide us with humanistic knowledge of great value for the research in mathematics and science education. This is a theme the author has advocated for (See Lybeck, Marton and Säljö, 1976; Lybeck, 1977, 1978 a, 1978 b).

5 In Sweden, we are at present discussing what basic knowledge should be included when constructing of the new curriculum and new courses of study for the compulsory school. It is our opinion that the description categories for proportional reasoning is useful in this work. Here, we would like to call attention to Freudenthal's suggestion of a vertical curriculum development based on ratio and proportion (Freudenthal, 1978). An integration of the school subjects, and the experiences outside the world of the school can be elaborated using our description categories as an instrument. Apart from a lot of research work, a large amount of developmental work will have to be done. With the aid of mathematics, the scientists are building models and theories. In fact, proportional reasoning is used in the model building thought patterns both inside and outside the world of the natural sciences.

We agree with Strauss (1977);

> "Thus, curriculum developers who decide on the types of formalizations of concepts to be taught in schools rarely take into account the students' intuitive, common sense notations about these concepts."

We hope that they will be more sensitive to the autonomous field of study, which has as its object of study the thought content of the students and adults in relation to some subject matter content. We want to emphasize the relational characteristics of those phenomena as have been done too by our colleagues Marton and Säljö (1978).

ACKNOWLEDGEMENT

The figures 1 - 4 are based on figures in Suarez (1974). We have used the same type of measuring cylinder as Suarez, supplied by the Limmat-Stiftung in Zürich.

The study of proportional reasoning is based on an internal report by Lybeck and Johansson (1977), and a report by Johansson and Lybeck (1978). The theoretical and empirical research was carried out during the fiscal years 1975/77 by Lybeck. During the fiscal year 1977/78 Johansson worked as a part-time assistant analysing the data on the Measuring Cylinder and the Spring Balance Task together with Lybeck.

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